Applications of minimax theorems

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In my lecture, I will give a survey of selected applications of certain minimax theorems that I have established in the past years.

Here is a sample:

THEOREM. - *Let X be a topological space and let $f, g : X \to \mathbf{R}$ be two functions such that, for each $\lambda > 0$, the function $f + \lambda g$ is sequentially lower semicontinuous and inf-sequentially compact.*

Then, at least one of the following assertions holds:

(i) f has at least one global minimum .

(ii) There exists $\lambda^* > 0$ such that the function $f + \lambda^* g$ has at least two global minima .

(iii) For each $r \in]\inf_X g, \sup_X g[$, there exists $\lambda_r > 0$ such that the function $f + \lambda_r g$ has a unique global minimum, $\hat{x}_r$, and one has $g(\hat{x}_r) = r$; in particular, $\hat{x}_r$ is the only global minimum of $f_{|g^{-1}(]-\infty,r])}$, each minimizing sequence for $f_{|g^{-1}(]-\infty,r])}$ converges to $\hat{x}_r$, and the functions $r \to \hat{x}_r$ and $r \to f(\hat{x}_r)$ are continuous in $]\inf_X g, \sup_X g[$.